AMERICAN UINVERSITY OF BEIRUT
FACULTY OF ENGINEERING AND ARCHITECTURE
EECE 460 FALL 2004-2005
Control Systems
Quiz I SOLUTION

Name:
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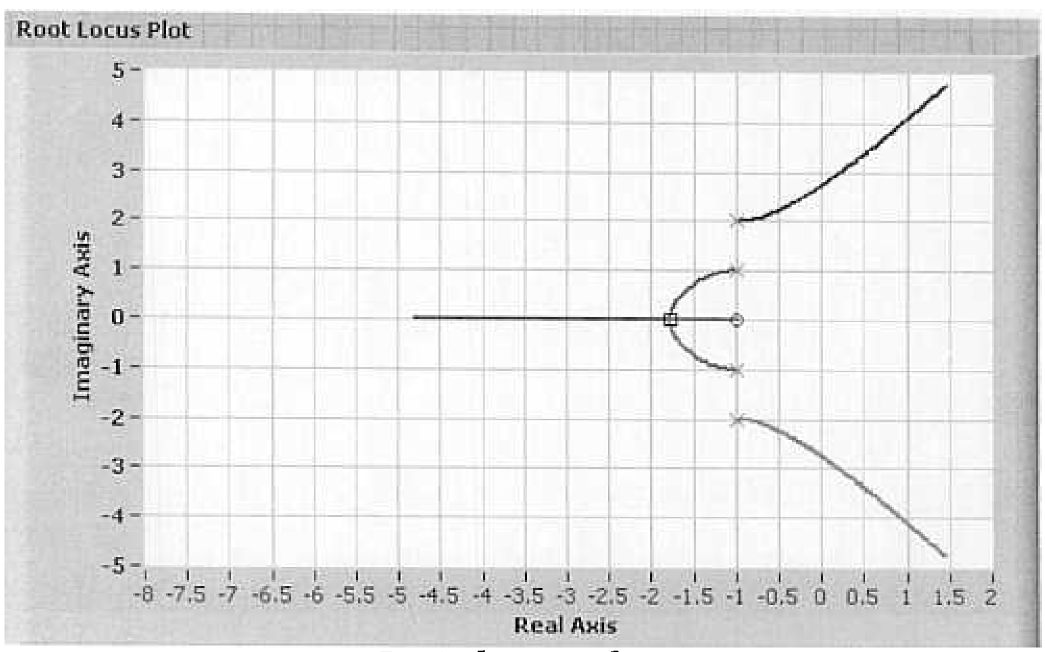
1.5 hours. Total of 100 points
Nov 2, 2004 Open Book Exam, 2 pages
YOU MUST RETURN THIS EXAM WITH YOUR ANSWER BOOKLET

Problem 1: (35 pts)

Consider the system whose open-loop transfer function $G(s)H(s)$ is given by

$$G(s)H(s) = \frac{K(s+1)}{(s^2+2s+2)(s^2+2s+5)}$$



Root-locus plot

+5 . Supply open loop transfer function poles for K = 20, and for that gain is open loop transfer function stable? Explain.
OLTF poles are from plot: -1 +/- 2j; -1+/- j oltf is stable (LHP poles)

+5 2. For positive K, characterize the stability of the closed loop system.
From root locus plot, system changes stability class with values of K, hence system is conditionally stable.

3. Approximate the value of K that makes the closed loop system marginally stable.
CLTF is marginally stable when locus crosses jw axis, approximately with 2 poles locates at +/- 2.8j, substitute one value in characteristic polynomial yields K = 14.74

+10

4 Approximate the range of K that makes the closed loop system absolutely stable
*CLTF is absolutely stable iff o<k<*14.7*from part 3 above.*

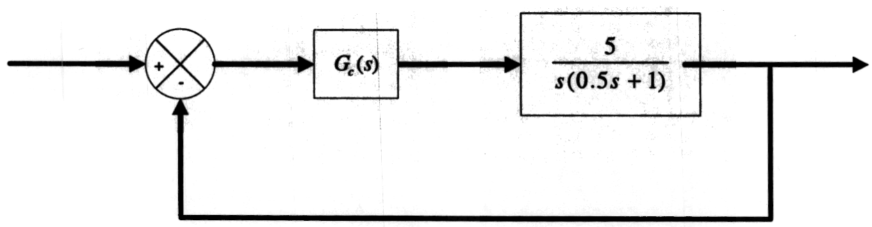
+5

5. Compute the value(s) of the gain corresponding to double poles and supply the double pole value (s).
The double poles occur based on locus plot at s= -1.75, substitute in characteristic polynomial and compute K = 9.5

+0

Problem 2: (40 pts)

Consider the unity feedback system open loop transfer function given by Gc(s)G(s)as shown below



1 Approximate manually the root locus of the closed loop system assuming that the controller is a P controller with gain K.

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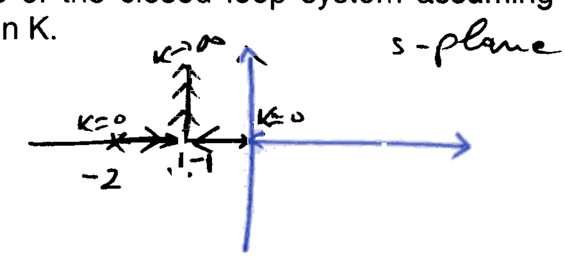
$G_c(s) = K$



2 Design (if possible) the proper P controller gain K, such that the dominant closed-loop poles are at desired locations: $s = -2 \pm j2\sqrt{3}$.
Desired poles are outside the locus, hence it is impossible to meet desired specs with P controller.

+5

3. What are the desired step-response maximum overshoot and settle time.
Desired poles are $s = -2 \pm j2\sqrt{3}$ *, hence desired characteristic poly is (s-s1d)(s-s2d) yields desired damping ratio and natural freq w, which yields corresponding desired Maximum overshoot* 16% *and Settle time* 2 sec

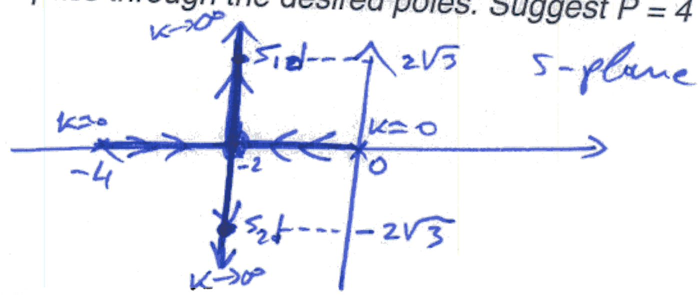
+7

4. I suggest the following controller composed of K positive gain, a pole (p) and a zero (z).

$$G_c(s) = K\frac{s+z}{s+p}$$

Propose a valid set of values for p and z (if possible) in order for the compensated (controlled) system root locus passes through the desired poles.
Z was supplied as 2 hence we only need to design P, we notice that the proposed zero cancels an open loop pole at –2, therefore one possible P would be to force the locus to pass through the desired poles. Suggest P = 4

+10



5. Supply the value (s) of K that meets the desired specs.
At desired pole location, substitute in the new characteristic polynomial and solve for K = 1.6

$$G_c(s) = K\frac{s+2}{s+4}$$

The gain K is determined from the magnitude condition.

$$\left|K\frac{s+2}{s+4}\frac{5}{s(0.5s+1)}\right|_{s=-2+j2\sqrt{3}} = 1$$

+10

or

$$K = \left|\frac{s(s+4)}{10}\right|_{s=-2+j2\sqrt{3}} = 1.6$$

Hence,

$$G_c(s) = 1.\ \frac{s+2}{s+4}$$

Problem 3: (25 pts)

Consider a linear system with multiple inputs and multiple outputs modeled by the state model:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -25 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

This system involves two inputs and two outputs.

1. Determine the four transfer functions:
   $Y_1(s)/U_1(s)$, $Y_2(s)/U_1(s)$, $Y_1(s)/U_2(s)$, $Y_2(s)/U_2(s)$

$$\frac{Y_1(s)}{U_1(s)} = \frac{s+4}{s^2+4s+25} \qquad + 4$$

$$\frac{Y_2(s)}{U_1(s)} = \frac{-25}{s^2+4s+25} \qquad + 4$$

$$\frac{Y_1(s)}{U_2(s)} = \frac{s+5}{s^2+4s+25} \qquad + 4$$

$$\frac{Y_2(s)}{U_2(s)} = \frac{s-25}{s^2+4s+25} \qquad + 4$$

2. If the inputs are both unit step, what is Y1(s)?

+ 5 *Y1(s) = [Y1(s)/U1(s)] 1/s + [Y1(s)/U2(s)] 1/s using above transfer functions*

3. If second input U2(s) is zero and U1(s) is unit impulse, what is Y2(s)?

+ 4 *Y2(s)= [Y2(s)/U1(s)] as supplied transfer function*

Hint: When considering input u_1, we assume that input u_2 is zero and vice versa since system is linear and we can apply superposition.